Cosmic acceleration from second order gauge gravity

R. R. Cuzinatto[1], C. A. M. de Melo[1,2], L. G. Medeiros[3], and P. J. Pompeia[4,5]

[1]*Instituto de Física Teórica, Universidade Estadual Paulista.*
Rua Pamplona 145, CEP 01405-900, São Paulo, SP, Brazil.

[2]*Universidade Vale do Rio Verde de Três Corações,*
Av. Castelo Branco, 82 - Chácara das Rosas,
P.O. Box 3050, CEP 37410-000, Três Corações, MG, Brazil

[3]*Centro Brasileiro de Pesquisas Físicas.*
Rua Xavier Sigaud 150, CEP 22290-180, Rio de Janeiro, RJ, Brazil

[4]*Comando-Geral de Tecnologia Aeroespacial,*
Instituto de Fomento e Coordenação Industrial.
Praça Mal. Eduardo Gomes 50, CEP 12228-901,
São José dos Campos, SP, Brazil. and

[5]*Theoretical Physics Institute, University of Alberta,*
Edmonton, Alberta, Canada, T6G 2J1.

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Abstract

We construct a phenomenological theory of gravitation based on a second order gauge formulation for the Lorentz group. The model presents a long-range modification for the gravitational field leading to a a cosmological model provided with an accelerated expansion at recent times. We fit the model parameters – such as the coupling constant of Lagrangian's sector scaling with the square of ∇R – using super-novae type Ia and X-ray galaxy clusters data. There is good agreement between our estimative for the age of the universe and the one predicted by the standard model. The transition from the decelerated expansion regime to the accelerated one occurs recently enough to indicate a solution of the cosmic coincidence problem.

I. INTRODUCTION

To explain the origin and evolution of the present accelerated expansion of the universe is one of the most challenging problems of Physics nowadays. One way to obtain a mechanism of acceleration is changing one of the cornerstones of modern physics, the theory of General Relativity.

Modifications in the scheme of General Relativity are being proposed since its invention, in the beginning of the 20th century, and they are motivated by several reasons, from the quest to agreement with the theory for the inner structure of quantized matter to eventual need of extra-dimensions and the desire to achieve the unification of interactions. The first modification of General Relativity was proposed by Einstein through the introduction of the cosmological constant, which is one of the several alternatives to describe the present acceleration of the universe. Other proposals in the context of Cosmology involve the introduction of one (or more) spatial extra-dimension in the braneworld scenario [1], or the presence of a self-interacting scalar field, the quintessence models [2].

Here, we shall explore some of the cosmological consequences of a phenomenological theory of gravitation based on a Lagrangian analyzed in [3]; we will show that this theory permits a recent accelerated phase for the universe without the introduction of Λ, exotic matter, extra-dimensions or scalar fields. It was constructed on the basis of a gauge formulation for the gravitational field, through the second order gauge theory [4]. This theory, by its turn, was shown to be efficient in the description of effective limits of other gauge theories, as Podolsky electrodynamics and the $SU(N)$ non-abelian model.

Our model uses differential equations of order higher than two in the metric tensor, but it is not equivalent to the usual approaches quadratic in the curvature tensor [5]. It is also not equivalent to the $f(R)$ theories discussed, for instance, in [6] and [7]. From the gauge theoretical point of view, quadratic Lagrangians in the Riemann tensor are actually of first order, and a classification of all possible quadratic Lagrangians of first and second order in the gauge gravitational field are given in [3]. Among these possibilities, we choose a particularly simple one, which is inspired by the Podolsky's abelian case and by the effective Alekseev-Arbuzov-Baikov's non-abelian model [8]. Actually, we choose to add a term scaling with the square of the covariant derivative of the scalar curvature ∇R to the familiar R of the Einstein-Hilbert Lagrangian. Because of this choice, our study is not the most general

one: we do not take a linear combination of all the gauge invariant Lagrangians presented in [3]. Our main intention is to analyze the consequences of this option in the cosmological context, and in particular if higher order derivative terms could produce interesting effects – such as the present-day cosmic acceleration of the scale factor.

We will adopt the following action for the description of the gravitational field:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\chi} + \frac{\beta}{\chi}\mathcal{L}_P - \mathcal{L}_M\right) \ , \tag{1}$$

$$\mathcal{L}_P = \frac{1}{8}\nabla_\mu R \nabla^\mu R \ . \tag{2}$$

Notice our option of not including the cosmological constant term in the action. At this stage this is only a simplicity hypothesis, but – we repeat – our final goal is to reproduce the present-day cosmic acceleration effect produced by Λ without imposing its existence *a priori*. The final results will show if our choice is paid of. It is worth to emphasize that the choice (2) constitutes a phenomenological model valid within a limited interval of energy (set by the values of the coupling constant β); it does not hold during all the cosmological history (as we shall see) but only for a certain period. The same phenomenological Lagrangian was applied to describe inflation in Ref. [9], but we emphasize the high order of magnitude of the energies involved there, which would correspond to the early stages of evolution of the Universe. On the other hand, the model presented here engenders sensible dynamical effects at recent cosmic times, where the energy scale is very low.

The paper is organized as follows. In section II the field equations are written for a Friedmann-Lemaître-Robertson-Walker metric. Section III is devoted to obtain a perturbative solution of the field equations about the usual dust-matter model of the Einstein-Hilbert theory (as described by the Friedmann equations). The perturbative solution is constructed in such a way that the universe is dominated by a decelerated regime until the time t^* from which the additional term $\frac{\beta}{\chi}\mathcal{L}_P$ begins to be relevant. In section IV the parameters of the model are related to the observational data, through a set of coupled nonlinear equations. Such equations are solved by numerical methods in section V, and the results are discussed in section VI.

II. FRIEDMANN EQUATIONS

The invariance of the action (1) with respect to $\delta g_{\lambda\nu}$ yields the field equations:

$$R_{\lambda\nu} - \frac{1}{2} g_{\lambda\nu} R + \beta^2 H_{\lambda\nu} = \chi T_{\lambda\nu} \ , \tag{3}$$
$$H_{\lambda\nu} = \nabla_\lambda \nabla_\nu \left[\Diamond R \right] + \frac{1}{2} \nabla_\lambda R \nabla_\nu R - R_{\lambda\nu} \Diamond R - g_{\lambda\nu} \Diamond \left[\Diamond R \right] - \frac{1}{4} g_{\lambda\nu} \nabla^\rho R \nabla_\rho R \ .$$

where $\Diamond \equiv \nabla_\mu \nabla^\mu$ and ∇_μ is the covariant derivative.

Applying the field equations (3) to a homogeneous and isotropic space, described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric,

$$ds^2 = dt^2 - a^2(t) \left(\frac{1}{1 - \kappa r^2} dr^2 + r^2 d\Omega^2 \right)$$

$(\kappa = -1, 0, +1)$ one finds, after some direct but long calculations,

$$-3\left(\dot{H} + H^2\right) - \frac{1}{2} R + \beta \left[-3H\dddot{R} + 3\dot{H}\ddot{R} - 6H^2\ddot{R} + 9H^3\dot{R} + \frac{1}{4}\dot{R}^2 \right] = \chi T_{00} \ ,$$
$$\frac{a^2}{(1-\kappa r^2)} \left[\dot{H} + 3H^2 + \frac{1}{2}R + 2\frac{\kappa}{a^2} + \beta \left(\ddddot{R} + 5H\dddot{R} + 3H^2\ddot{R} + 5\dot{H}\ddot{R} + \right.\right.$$
$$\left.\left. + 3H\dot{H}\dot{R} - 9H^3\dot{R} + 3\ddot{H}\dot{R} + \frac{1}{4}\dot{R}^2 - 2\frac{\kappa}{a^2}\left(\ddot{R} + 3H\dot{R}\right) \right) \right] = \chi T_{11} \ . \tag{4}$$

where $H(t) = \dot{a}/a$ is the Hubble function, $R(t) = g^{\mu\nu} R_{\mu\nu}$ is the scalar curvature and we are using units such that $\chi = 8\pi G$. In our notation, dot means derivation with respect to the cosmic time t. These are the higher order Friedmann equations in terms of the Hubble function $H(t)$ and the scalar curvature $R(t)$.

Following the standard procedure we use the energy-momentum tensor of a perfect fluid in a commoving coordinate system,

$$T_{\mu\nu} = (\rho + p)\, \delta^0_\mu \delta^0_\nu - p g_{\mu\nu} \ .$$

In order to simplify the treatment, we will be concerned only with the case of a flat spatial section, $\kappa = 0$. So, using the relationship between the scalar curvature and the Hubble function,

$$R = -6\left(\dot{H} + 2H^2\right) \ ,$$

we get the following modified Friedmann equations:

$$
\begin{aligned}
&3H^2 + \beta \Big(18H \ddddot{H} + 108H^2 \dddot{H} - 18\dot{H}\dddot{H} + 9\ddot{H}^2 + 90H^3\ddot{H} + \\
&\quad +216H\dot{H}\ddot{H} - 72\dot{H}^3 + 288\left(H\dot{H}\right)^2 - 216H^4\dot{H} \Big) = \chi\rho \ , \\
&2\dot{H} + 3H^2 + \beta \Big(6H^{(5)} + 54H\ddddot{H} + 138H^2\dddot{H} + 126\dot{H}\dddot{H} + \\
&\quad +81\ddot{H}^2 + 18H^3\ddot{H} + 498H\dot{H}\ddot{H} + 120\dot{H}^3 - 216H^4\dot{H} \Big) = -\chi p \ .
\end{aligned}
$$

Combining the equations, one finds:

$$
\begin{aligned}
&2\dot{H} + \beta \Big(6H^{(5)} + 36H\ddddot{H} + 30H^2\dddot{H} + 144\dot{H}\dddot{H} + 72\ddot{H}^2 + \\
&\quad -72H^3\ddot{H} + 282H\dot{H}\ddot{H} + 192\dot{H}^3 - 288\left(H\dot{H}\right)^2 \Big) = -\chi\left(p+\rho\right) \ .
\end{aligned}
\tag{5}
$$

As we want to describe the evolution of the universe, this equation must be complemented with the covariant conservation of energy-momentum,

$$\dot{\rho} + 3H\left(\rho + p\right) = 0 \ ,$$

and an equation of state f relating the energy density ρ, the pressure p and the Hubble function H,

$$f\left(\rho, p, H\right) = 0 \ .$$

The dependence on H is included to account for the general case when one admits interaction among the constituents of the cosmic fluid [11]. In this case, there is a possible constraint relating p, ρ and the scale factor, or equivalently H. On the other hand, the usual equations of state of physical cosmology associate only pressure p to the energy density ρ, or pressure to the numerical density n. For example, the equation of state for the dust matter is $p = nkT \ll \rho$ (k is the Boltzmann constant and T the temperature), and $p = \rho/3$ is the one used for ultra-relativistic particles.

III. SOLUTIONS OF THE HIGHER ORDER FRIEDMANN EQUATIONS

A. Dust Matter

Our main interest here is to apply the model to the present state of the universe. Therefore, we take as source a perfect fluid composed by dust matter $p = 0$ (ordinary or dark).

In this case, the continuity equation gives:

$$\rho\left(t\right)=\rho_0\left(\frac{a_0}{a\left(t\right)}\right)^3 \ .$$

In order to use such result directly, we would have to rewrite equation (5) in terms of the scale factor, obtaining a nonlinear and much more complicate equation, which we shall avoid. Instead, we consider simultaneously the following pair of coupled equations:

$$\begin{aligned}
&\dot{H}+\beta\Big(3H^{(5)}+18H\ddddot{H}+15H^2\dddot{H}+72\dot{H}\dddot{H}+36\ddot{H}^2+\\
&-36H^3\ddot{H}+141H\dot{H}\ddot{H}+96\dot{H}^3-144\left(H\dot{H}\right)^2\Big)=-\frac{\chi}{2}\rho \ , \qquad (6)\\
&\dot{\rho}+3H\rho=0 \ .
\end{aligned}$$

These equations can be analyzed by several methods, such as linearization for dynamical systems, spectral analysis or perturbation theory. Here, we will consider only this last procedure, leaving the other options for future investigations.

B. Perturbation Theory

The model is constructed by assuming a standard Friedmann expansion prior to some time t^* from which the second order effects start to become significant. The strategy is to consider a perturbation series in the coupling parameter β, in order to guarantee the accordance of our model with the usual cosmological model (in some region of the space of parameters). Take, for instance, an expansion up to second order terms; it reads:

$$\begin{aligned}
H\left(t\right)&=H_F+\beta H_1+\beta^2H_2 \ ,\\
\rho\left(t\right)&=\rho_F+\beta\rho_1+\beta^2\rho_2 \ ,
\end{aligned} \qquad (7)$$

where the label F stands for the standard Friedmann solution of the Einstein equations.

Substituting expansions (7) in the pair (6) and matching the terms order by order, we get:

$$\mathcal{O}\left(\beta^0\right)\rightarrow\begin{cases}\dot{H}_F+\frac{\chi}{2}\rho_F=0\\ \dot{\rho}_F+3H_F\rho_F=0\end{cases} \ , \qquad (8)$$

and,

$$\mathcal{O}\left(\beta^1\right)\rightarrow\begin{cases}\dot{H}_1+\frac{\chi}{2}\rho_1=S_1\left(t\right)\\ \dot{\rho}_1+3H_F\rho_1+3H_1\rho_F=0\end{cases} \ , \qquad (9)$$

where

$$S_1(t) \equiv -\Big(3H_F^{(5)} + 18H_F \ddddot{H}_F + 15H_F^2 \dddot{H}_F + 72\dot{H}_F \dddot{H}_F + 36\ddot{H}_F^2 \\ -36H_F^3 \ddot{H}_F + 141H_F \dot{H}_F \ddot{H}_F + 96\dot{H}_F^3 - 144H_F^2 \dot{H}_F^2\Big) \;; \tag{10}$$

and also,

$$\mathcal{O}\left(\beta^2\right) \rightarrow \begin{cases} \dot{H}_2 + \frac{\chi}{2}\rho_2 = S_2(t) \\ \dot{\rho}_2 + 3H_F\rho_2 + 3H_2\rho_F = -3H_1\rho_1 \end{cases} , \tag{11}$$

with,

$$\begin{aligned} S_2(t) \equiv &-\Big[3H_1^{(5)} + 18\left(H_1 \ddddot{H}_F + H_F \ddddot{H}_1\right) + 30H_F H_1 \dddot{H}_F + \\ &+72\left(\dot{H}_F \dddot{H}_1 + \dot{H}_1 \dddot{H}_F + \ddot{H}_F \ddot{H}_1\right) - 108H_1 H_F^2 \ddot{H}_F - 36H_F^3 \ddot{H}_1\Big] + \\ &-\Big[141\left(H_1 \dot{H}_F \ddot{H}_F + H_F \dot{H}_F \ddot{H}_1 + H_F \dot{H}_1 \ddot{H}_F\right) + 288\left(\dot{H}_1 \dot{H}_F^2 - H_F^2 \dot{H}_F \dot{H}_1 - H_F H_1 \dot{H}_F^2\right)\Big] \;. \end{aligned} \tag{12}$$

This way, we obtained a pair of coupled linear equations in each order. Their previous orders give the source term and the coefficients.

1. *Zeroth order solution: the standard cosmological model*

The solution for the system of zeroth order in the coupling parameter β, Eq. (8),

$$\dot{H}_F + \frac{\chi}{2}\rho_F = 0 \;, \tag{13}$$

$$\dot{\rho}_F + 3H_F\rho_F = 0 \;, \tag{14}$$

can be obtained by direct integration. Substituting (13) in (14), we have:

$$\rho_F = \frac{3}{\chi}H_F^2 + k_1$$

where k_1 is an integration constant. Substituting this back into (13) and choosing the initial condition $k_1 = 0$,

$$\frac{1}{H_F} = \frac{3}{2}t + k_2 \;.$$

Setting $k_2 = 0$, we obtain the standard Friedmann solution for $H(t)$:

$$H_F = \frac{2}{3}\frac{1}{t} \;. \tag{15}$$

2. First order solution

In the first order approximation, we have the coupled set (9),

$$\dot{H}_1 + \frac{\chi}{2}\rho_1 = S_1\left(t\right) \ ,$$
$$\dot{\rho}_1 + 3H_F\rho_1 + 3H_1\rho_F = 0 \ ,$$

These equations can be solved by the Increasing Order Method. Differentiating the first of these equations and using the second one, we obtain

$$\ddot{H}_1 + 3H_F\dot{H}_1 - \frac{3\chi}{2}\rho_F H_1 = \tilde{S}_1\left(t\right)$$
$$\tilde{S}_1\left(t\right) = \dot{S}_1\left(t\right) + 3H_F S_1\left(t\right)$$

The general solution of such equation can be obtained in the form of a power law:

$$H_1\left(t\right) = at + bt^{-2} + \frac{4912}{243}t^{-5}$$
$$\rho_1\left(t\right) = \frac{2}{\chi}\left(-\frac{15\,977}{243}t^{-6} - a + 2bt^{-3}\right)$$

The integration constants a and b should be chosen in accordance to the physical situation to be described. Since the zeroth order terms appear as source terms in the first order approximation, one can choose the initial conditions $H_1\left(t^*\right) = \rho_1\left(t^*\right) = 0$. This determine the integration constants leaving the theory with only three free parameters, namely the coupling constant β, the age of the universe t_0 (see below) and the instant of perturbation t^*.

Therefore, in the first order approximation we find the following solution to equations (6):

$$H\left(t\right) = \frac{2}{3}\frac{1}{t} + \frac{\beta}{\left(t^*\right)^4}\left(\frac{11\,065}{729}\left(\frac{t^*}{t}\right)t^{-1} + \frac{4912}{243}\left(\frac{t^*}{t}\right)^4 t^{-1} - \frac{35\,408}{729}\frac{t}{t^*}\left(t^*\right)^{-1}\right)$$
$$8\pi G\rho\left(t\right) = \frac{4}{3}\frac{1}{t^2} + 2\frac{\beta}{\left(t^*\right)^4}\left(\frac{22\,130}{729}\left(\frac{t^*}{t}\right)t^{-2} - \frac{15\,977}{243}\left(\frac{t^*}{t}\right)^4 t^{-2} + \frac{35\,408}{729}\left(t^*\right)^{-2}\right) \ . \quad (16)$$

IV. OBSERVATIONAL PARAMETERS

Now, let us focus on the problem of linking our theoretical model with the observational data available. Once the main goal of the present model is to reproduce the present accel-

eration without using the cosmological constant or scalar fields, we must choose the data which are model-independent.

The redshift z and the luminosity distance d_L are dependent on the null geodesic equation only, and this is not changed by the second order field equations (3). Therefore, they constitute the ideal data set to be compared with the predictions of our model. The luminosity distance can be directly related to the redshift [12],

$$d_L \approx \frac{1}{H_0}\left(z + \frac{1}{2}\left(1 - q_0\right) z^2\right) .$$

q_0 is the deceleration parameter.

The supernovae projects usually measure the curve of $d_L\left(z\right)$ determining the parameters H_0 and q_0 with good accuracy. However, in our model we have three parameters to be found:

1. The age of the universe t_0;
2. The instant t^* from which the perturbation coming from the modified gravitational equation becomes important;
3. The coupling constant β for the higher derivative terms in the action.

We need three independent measurements to find these parameters. Two of them are naturally given by the supernovae experiments – through the values of H_0 and q_0. The third is suggested by the very conservation equation for the energy momentum tensor. Since the presented model considers only dust matter, we need the value of the total amount of matter in the universe, or equivalently its ratio with respect to the usual critical density, Ω_{m0}. This information can be provided by X-ray experiments measuring the gas fraction in galaxy clusters [13]. The advantage of using the results of this particular measurement is the fact that it is completely independent of the model for the evolution of the universe. This is not the case for the more common WMAP data: they assume the validity of ΛCDM model.

In the following section, we shall carefully discuss how to use H_0, q_0 and Ω_m to obtain t_0, t^* and β. But, before that, we will add to system (16) the constraint

$$\dot{H}\left(t_0\right) = -H_0^2\left(q_0 + 1\right) \tag{17}$$

following from the definition of both the Hubble and the deceleration functions in terms of the scale factor: $H = \dot{a}/a$, $q = -\ddot{a}a/\dot{a}^2$. Gathering (16) and (17), we get the new system to

be solved:

$$H\left(t_0\right)=\frac{2}{3}\frac{1}{t_0}+\frac{\beta}{\left(t^*\right)^4}\left(-\frac{35\,408}{729}\frac{t_0}{t^*}\left(t^*\right)^{-1}+\frac{11\,065}{729}\left(\frac{t_0}{t^*}\right)^{-1}t_0^{-1}+\frac{4912}{243}\left(\frac{t_0}{t^*}\right)^{-4}t_0^{-1}\right)$$

$$\dot{H}\left(t_0\right)=-\frac{2}{3}\frac{1}{t_0^2}+\frac{\beta}{\left(t^*\right)^4}\left(-\frac{35\,408}{729}\left(t^*\right)^{-2}-\frac{22\,130}{729}\left(\frac{t_0}{t^*}\right)^{-1}t_0^{-2}-\frac{24\,560}{243}\left(\frac{t_0}{t^*}\right)^{-4}t_0^{-2}\right)$$

$$8\pi G\rho\left(t_0\right)=\frac{4}{3}\frac{1}{t_0^2}+2\frac{\beta}{\left(t^*\right)^4}\left(-\frac{15\,977}{243}\left(\frac{t_0}{t^*}\right)^{-4}t_0^{-2}+\frac{35\,408}{729}\left(t^*\right)^{-2}+\frac{22130}{729}\left(\frac{t_0}{t^*}\right)^{-1}t_0^{-2}\right)$$

The first member of each equation of the system above is given in terms of observational constants, while the right hand side of each equality bears the parameters of the perturbed model. These will be calculated by solving numerically the above transcendental equations. We deal with this task now.

V. NUMERICAL CALCULATIONS

In order to solve numerically the system of coupled transcendental equations, let us perform some simple manipulations. First, we define new non-dimensional variables,

$$u\equiv\frac{t^*}{t_0}\ ,\ b\equiv\frac{\beta}{t^{*4}}\ ; \tag{18}$$

in terms of which the system is rewritten as

$$H_0=\frac{2}{3}\frac{1}{t_0}\left(1+b\left(-\frac{17\,704}{243}\frac{1}{u^2}+\frac{11\,065}{486}u+\frac{2456}{81}u^4\right)\right)\ ; \tag{19}$$

$$H_0^2\left(q_0+1\right)=\frac{2}{3}\frac{1}{t_0^2}\left(1+b\left(\frac{17\,704}{243}\frac{1}{u^2}+\frac{11\,065}{243}u+\frac{12\,280}{81}u^4\right)\right)\ ; \tag{20}$$

$$3H_0^2\Omega_{m0}=\frac{4}{3}\frac{1}{t_0^2}\left(1+b\left(-\frac{15\,977}{162}u^4+\frac{17\,704}{243}\frac{1}{u^2}+\frac{11\,065}{243}u\right)\right)\ , \tag{21}$$

where

$$\Omega_{m0}\equiv\frac{8\pi G}{3H_0^2}\rho_0\,.$$

Taking the ratio of the two last equations of the system above, one gets:

$$b\left(u\right)=\frac{2\left(q_0+1\right)-3\Omega_{m0}}{s\left(u\right)}\,, \tag{22}$$

$$s\left(u\right)\equiv\left(3\Omega_{m0}-2\left(q_0+1\right)\right)\left(\frac{17\,704}{243}\frac{1}{u^2}+\frac{11\,065}{243}u+\frac{36\,840}{243}u^4\right)+\frac{40\,537}{81}\left(q_0+1\right)u^4\,.$$

We must have $\beta < 0$ in order to assure the stability of the theory [3, 9]. This establishes a constraint on the sign of b. Substituting $b\,(u)$ in the equation for H_0:

$$\frac{1}{t_0\,(u)} = \frac{3}{2}\frac{H_0}{\left(1 + b\,(u)\left(-\frac{17\,704}{243}\frac{1}{u^2} + \frac{11\,065}{486}u + \frac{2456}{81}u^4\right)\right)}\,. \tag{23}$$

Combining (19) and (20) we get

$$H_0^2\,(2\,(q_0+1) - 3\Omega_{m0}) = \frac{4}{3}\frac{b\,(u)}{t_0^2\,(u)}\left(\frac{12\,280}{81} + \frac{15\,977}{162}\right)u^4\,,$$

which is a nonlinear equation for the parameter u. This equation can be solved using the Newton-Raphson method. The result is then used in (22) to obtain the numerical value of b and both b and u are inserted in (23) to get the age of the universe t_0. The time of perturbation t^* can then be obtained as a simple ratio. Proceeding this way, we are able to find all the parameters of the model given the measurements of H_0, q_0 and ρ_0 (or Ω_{m0}). Notwithstanding, such measurements have uncertainties, which are propagated to the parameters of our model with a confidence level of 68% (one σ). This involves a problem of nonlinear programming [17] using the algorithm described above implemented in the software *Mathematica*® *6.0*.

With this assumptions and using the experimental values [13, 14]

$$\begin{aligned} H_0 &= 0.074^{+0.007}_{-0.007}\ (Gyr)^{-1}\ , \\ q_0 &= -0.81^{+0.14}_{-0.14}\,, \\ \Omega_{m0} &= 0.35^{+0.06}_{-0.06}\ , \end{aligned}$$

we find:

$$\begin{aligned} u &= 0.80^{+0.06}_{-0.06}\,, \\ b &= -0.0037^{+0.0007}_{-0.0007}\,, \\ t^* &= 9.4^{+1.4}_{-1.4}\ Gyr\ , \end{aligned}$$

which can be used to calculate the parameter β and the age of the universe,

$$\beta = -29^{+17}_{-32}\ (Gyr)^4\ , \tag{24}$$

$$t_0 = 11.8^{+1.3}_{-1.4}\ Gyr\ . \tag{25}$$

In spite of its high value, β does not break the meaning of the modified action as proposed in (1). This is so because the own curvature tensor (and its derivatives) is β-dependent, such

that the additional term $L_P = \frac{\beta}{8}\sqrt{-g\left(\beta\right)}\nabla_\mu R\left(\beta\right)\nabla^\mu R\left(\beta\right)$ scales as β/t^{*4} and therefore is small when compared to the usual Einstein-Hilbert term. The fact that the uncertainty in β is very large – it is of the same order of β – is not critical for our model precisely because $\beta/t^{*4} \ll 1$. Of course, the contribution of the additional term goes up for values of t progressively greater than t^*. But we do not hope that the perturbative approach holds for arbitrary large values of t. The convergence of the perturbative expansion can be qualitatively studied plotting the ratio of the perturbation term by the usual FLRW solution, as given in Fig. 1.

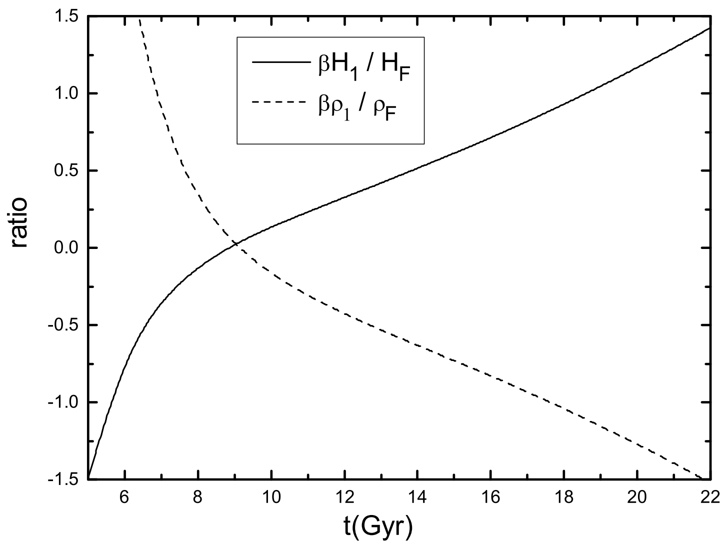

FIG. 1: Ratios of the first term in the perturbative solutions by the ordinary dust-matter solution as functions of the time. The full line shows the evolution of the the ratio of the first-order perturbed Hubble function divided by the non-perturbed one; the dashed line presents the behavior of the perturbation on the energy density divided by its non-perturbed values.

As long as the curves lie within the interval $[-1, 1]$ the perturbative scheme can be assumed as valid. Notice that the Hubble function fits the perturbative scheme for a longer time compared to the behavior of the energy density.

The value of t_0 provided by the WMAP3 data [16] – which assumes the ΛCDM model – is calculated as $13.7^{+0.1}_{-0.2}$ Gyr (for a flat Universe). The result given by our model is in agreement with the one predicted by the ΛCDM model at the 2σ-level. This apparent tension between both results does not impair the model proposed here since results obtained

from anisotropies are strongly dependent of the adjusted model, in this case the ΛCDM one. Besides, globular clusters data gives 11.2 Gyr, at 95% of confidence level, as inferior limit for the age of the universe [18].

A. The "Coincidence Problem"

Using the solution (16), one easily finds the ratio of scale factor at two arbitrary times t_i and t_f as

$$\ln\frac{a_f}{a_i} = \ln\left(\frac{t_f}{t_i}\right)^{\frac{2}{3}} - \frac{\beta}{(t^*)^4}\left(\frac{17\,704}{729}\left(\frac{t_f}{t^*}\right)^2 + \frac{11\,065}{729}\left(\frac{t_f}{t^*}\right)^{-1} + \frac{1228}{243}\left(\frac{t_f}{t^*}\right)^{-4}\right) + \\ + \frac{\beta}{(t^*)^4}\left(\frac{17\,704}{729}\left(\frac{t_i}{t^*}\right)^2 + \frac{11\,065}{729}\left(\frac{t_i}{t^*}\right)^{-1} + \frac{1228}{243}\left(\frac{t_i}{t^*}\right)^{-4}\right) ,$$

which exhibits a very smooth transition from the Friedmann standard regime to the accelerated one – see Figure bellow.

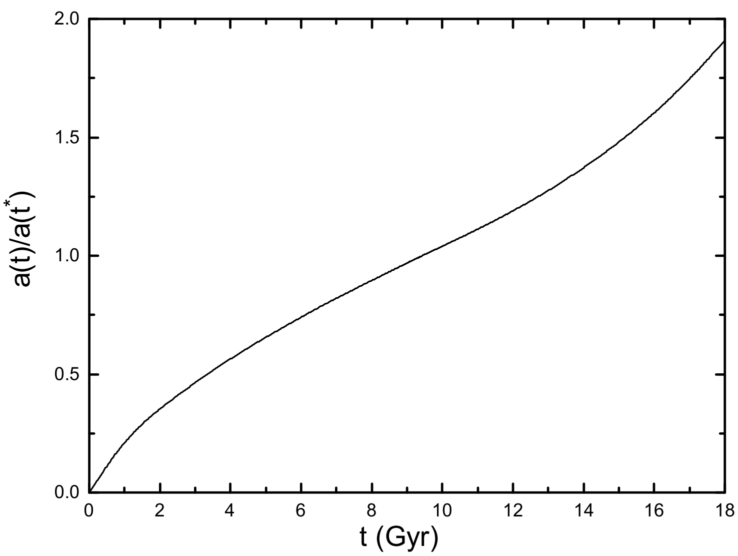

FIG. 2: Plot of the complete scale factor (matter solution plus higher order terms) as a function of time. Notice that the decelerated regime is roughly in the first half of the graphic; the curve shows clearly the onset of acceleration at recent times.

The same quantity can be used to estimate the red-shift at the transition,

$$1 + z^* = \frac{a_0}{a\,(t^*)} ,$$

as

$$1+z^* = \left(\frac{t_0}{t^*}\right)^{\frac{2}{3}} \times \exp\left(-\frac{\beta}{(t^*)^4}\left(\frac{17\,704}{729}\left(\frac{t_0}{t^*}\right)^2 + \frac{11\,065}{729}\left(\frac{t_0}{t^*}\right)^{-1} + \frac{1228}{243}\left(\frac{t_0}{t^*}\right)^{-4} - \frac{32\,453}{729}\right)\right) .$$

With the values of β, t^* and t_0, one calculates:

$$z^* = 0.19^{+0.07}_{-0.07} .$$

Therefore, our second order Lagrangian-based model presents a possible explanation for the "cosmic coincidence" problem, since the observed values of the present rate of expansion (H_0), acceleration (q_0) and total mass (Ω_{m0}) imply a transition from a decelerated phase to the present accelerated regime at a relatively low redshift.

VI. CONCLUSIONS

We have constructed a model based on a phenomenological theory of gravitation obtained from the inclusion of a Podolsky-like term scaling with the square of the covariant derivative of the Ricci scalar. This model implies long-range modifications in gravitation, which leads to an accelerated regime for the present-day universe, even in the absence of a dark energy component or cosmological constant. According to our perturbative evaluation, this accelerated expansion started recently as indicated by the values of t^* or, equivalently, z^*.

The values predicted by the model for the age of the universe and the redshift of transition are in agreement with the supernovae data [14] or the analysis of the cosmic microwave background based on the ΛCDM model [15] at the 2σ-level. We did not use the CMB data to predict values for the age of the universe, our intention being to avoid model-dependence. But these calculations could be done in the future. Our perspectives include the study of a perturbative solution for a closed ($\kappa = 1$) universe, keeping in mind the exact solution found in the ordinary FLRW case [19].

The fact that the gravitational field is weaker at long distances with a characteristic scale given by the coupling constant β suggests the existence of massive modes in the weak field approximation, but in a way that does not break the coordinate invariance, analogously to what happens in the Podolsky electrodynamics [4]. The eventual existence of such massive modes are under investigation, and the results should be compared to other approaches in the same direction [20].

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